

Programmable matter by folding

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Programmable matter is a material whose properties can be programmed to achieve specific shapes or stiffnesses upon command. This concept requires constituent elements to interact and rearrange intelligently in order to meet the goal. This paper considers achieving programmable sheets that can form themselves in different shapes autonomously by folding. Past approaches to creating transforming machines have been limited by the small feature sizes, the large number of components, and the associated complexity of communication among the units. We seek to mitigate these difficulties through the unique concept of self-folding origami with universal crease patterns. This approach exploits a single sheet composed of interconnected triangular sections. The sheet is able to fold into a set of predetermined shapes using embedded actuation. To implement this self-folding origami concept, we have developed a scalable end-to-end planning and fabrication process. Given a set of desired objects, the system computes an optimized design for a single sheet and multiple controllers to achieve each of the desired objects. The material, called programmable matter by folding, is an example of a system capable of achieving multiple shapes for multiple functions.

reconfigurable robotics | self-assembly | multifunctional materials | computational origami

Every day, scientists and engineers design new devices to solve a current problem. Each device has a unique function and thus has a unique form. The geometry of a cup is designed to hold liquid and is therefore different from that of a knife which is meant to cut. Even if both are made of the same material (e.g., metal, ceramic, or plastic), neither can perform both tasks. Is this redundancy in material, yet limitation in tasks entirely necessary? Is it possible to create a programmable material that can reshape for multiple tasks?

Programmable matter is a material whose properties can be programmed to achieve specific shapes or stiffnesses upon command. In this paper we consider the theory and design of programmable matter material that can assume multiple desired shapes on demand.

We have developed a unique concept of self-folding origami with universal crease patterns that decreases the complexity in individual elements and is scalable in the number and size of elements. Instead of relying on many individual subunits, which may be complex and difficult to orient correctly, we utilize a single sheet with repeated triangular tiles connected by flexible creases. This sheet can fold with a certain crease pattern to create multiple three-dimensional shapes, depending on which creases fold, in which direction, and in which order. We build on a large body of prior work in self-reconfiguring robotics to realize machines with changing shapes. Self-reconfiguring robots are modular systems whose bodies consist of multiple modules that can communicate and move relative to each other to form different shapes. These shapes support the different locomotive, manipulative, or sensing needs of the robot. Several different module designs and algorithms for coordinating the movement of these systems have been proposed. There are three classes of modular robots: chain (1), lattice (2), and hybrid (3–5). The modules move relative to each other using relative module motion (3, 5), module disconnection (6), or random motion (7, 8). The theoretical investiga-

tions consider motion planning and design bounds (9–11), generic planners that can be instantiated to different robot bodies (12), and architecture-specific planners. An extensive review of the achievements and challenges in modular self-reconfiguring machines is given by ref. 13. Much of this prior work considers unit-modules on the order of 5–10 cm in size, whereas our work involves 1 cm modules. The work in this paper departs from the principle of relative module motion. The self-folding sheets have underlying modularity provided by the crease pattern but the triangular modules do not change location relative to each other. The fixed sheet structure reduces complexity in design and planning and increases robustness by eliminating the need for continuously making and breaking connections to neighbors. Our work is related to recent work which has demonstrated the use of fluid forces to bend planar structures into well-defined three-dimensional structures (14). This system is analogous to our method of using actuation embedded within a surface to generate folds, however, without the ability to dynamically alter surface properties (e.g., dynamically controlled wetting properties), the user is restricted to a single shape programmed at design time.

To realize self-folding sheets, we follow an end-to-end fabrication process from a set of desired shapes to a self-folding sheet that can realize each shape autonomously on demand. This process consists of two steps: the first step implements four planning algorithms in sequence. These algorithms determine a plan for folding certain creases such that all goal shapes can be created from a single sheet. Second, we fabricate a sheet according to this plan: for fabrication, we utilize recent developments in multimaterial manufacturing, embedded actuation, and stretchable electronics. In *Process* we describe the physical instantiation of the sheet, and in particular the thermally induced solid state phase transition (shape memory) actuators. By implementing this two-step process, we have demonstrated working programmable matter.

In this report, we present the theory behind our origami concept, the planning and fabrication process, and finally the results of a fabricated sheet less than 0.5 mm thick that achieves two complex functional shapes, a *boat* and an *airplane*. Fig. 1 shows the current self-folding sheet, highlighting the key components. We also discuss the gaps between theory and practice, and point to a number of open questions toward more capable future programmable matter.

Theoretical Background

The theoretical basis of our self-folding programmable matter arises from the field of computational origami, an area of computer science that began in the 1990s (15, 16). One early result in this field (17) states that every polyhedral surface can be folded from a sufficiently large square of paper, and the folding can be computed in polynomial time. Thus a large enough sheet of

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The third algorithm receives each of the individual plans from the second algorithm and assembles them onto one sheet. Then the algorithm takes all creases that need to be folded from all the incoming plans and divides them into groups. These groups are created such that every phase from the incoming plans can be written as a linear combination of the new groups. That is, activating one or more of the groups created by the third algorithm can create the first, second, third, etc. phase from the plan for the first shape, as well as the first, second, third, etc. phase for the second goal shape, and so on. This algorithm is run once for each possible arrangement of individual plans from the second algorithm on a single sheet. For example, if there are two goal shapes and the second algorithm returns two corresponding plans, then these two plans can fit simultaneously on a single square sheet in eight different arrangements (four 90° rotations and one 180° flip).

Finally, the fourth algorithm chooses the optimum arrangement from the output of the third algorithm to minimize either the number of actuators or number of actuator groups. We either minimize the number of actuators to save energy and fabrication time, or minimize the number of groups of actuators to eliminate unnecessary circuitry and minimize folding time by maximizing parallelism. Finally, we check for detrimental antagonistic overlaps which can cause incorrect folding. An antagonistic overlap requires a crease to bend one way for one shape and the opposite way for a second shape. Bidirectional actuation is possible in practice but brings added mechanical complexity, which we avoid with the simple unidirectional actuators described as part of the fabrication process. Therefore, we remove the actuator from such a crease to allow passive motion caused by the folding of other creases. Often we find that the folding of other creases correctly folds the passive crease, both in simulation and in practice; otherwise, we call the passive crease detrimental. Any arrangement with a detrimental antagonistic overlap is discarded.

The running time of the last three algorithms is polynomial for a fixed number of target shapes. For a large number of shapes, we would have to replace the exhaustive search over all arrangements of individual plans on a sheet. The running time of the first algorithm has not been analyzed, but its similarity to the gradient flow of ref. 21 suggests that a pseudopolynomial time bound holds.

Fabrication Process. The second step of our process is the fabrication of the three components of the programmable material: the tiled composite sheet, thin-foil actuators, and flexible electronics. The first component, the sheet, must have semirigid tiles to transmit force from the actuators, yet have joints stretchable enough to allow compound folding*. To create the sheet we laminate a single sheet of sixteen-layer E-glass fiber (104 weave), impregnated with RS-30 resin. Glass-reinforced composites are used for the tiles because of high strength, light weight, ease of machinability, and material compatibility with silicone-based elastomers used for the joints. The uncured composite sheet is laser micromachined with a CO₂ laser (from Universal Laser Systems) to create slots for magnets. The magnets are used to create the potential wells discussed previously in *Theoretical Background*. The composite is cured at 140° C for four hours then micromachined to form the tile pattern. Alignment is ensured by machining the tiles while attached to a tacky substrate (Gel-Pak X8). Flexures are created by casting two-part GI-1100 RTV-2 silicone rubber (Fig. 1E). Elastomeric joints are essential for complex shapes since the joints need to stretch as well as bend. The resulting sheet can then be released and further machined for actuator attachment, metal deposition, or other additive or subtractive processes. This composite process shares similarities with Shape Deposition Manufacturing (22) and Smart Composite Microstructures (23), both

of which were established as rapid fabrication techniques for composite robotic structures.

Actuators for the sheet must be low profile, simple to attach to the sheet substrate, and easy to create in quantities of tens or hundreds (depending upon the sheet dimensions and resolution). To put the actuator requirements in perspective, relatively large torques are necessary (on the order of tens of millinewton-meters) to account for the mass of adjacent tiles, but the thickness cannot exceed that of the sheet (approximately 500 μm) so as to avoid interference during operation. High torques can be achieved geometrically (i.e., using a larger moment arm), however this is not a viable option for the sheets. Our unique thin-foil actuators, the second physical component, are capable of 180° of folding motion and are easily embedded within the tiling. We use thin (100 μm) foil Nitinol shape memory alloy (SMA) for the actuator material. The planform geometry is formed with a pulsed ultraviolet laser micromachining system. The micromachined foil is tinned, fixed in a jig to hold a desired (folded) shape, and annealed at 420° C for 30 min. This annealing process resets the undeformed martensitic state such that when unfolded, the actuator will “remember” its folded shape when heated above its transition temperature of 70° C (24). Thus any fold that is annealed into SMA can be manually flattened, and upon actuation, the fold is reformed. There are four folds in each actuator: one central bending fold, and three distal clamping folds. For assembly onto the sheet, we open all four folds (Fig. 3, *top*). The actuator is placed onto the sheet with each of the three legs inserting into a through-hole. The distal bends are heated with an external source, causing them to close (Fig. 3, *middle*). This results in a heat-activated staple, which tightly grips the substrate (Fig. 1A, B). When the central fold is heated, the actuator folds tightly closed, which is the source of folding actuation for the sheet (Fig. 3, *bottom*). These actuators only remember folds in one direction (i.e., they fold from 0–180°). We have also prototyped antagonistic (bidirectional) actuators, however, for the algorithms and shapes discussed in this paper, only unidirectional actuators are needed. Thus, to minimize undesired complexity and maximize fabrication efficiency, only unidirectional actuation is described.

The third component of programmable matter is the stretchable electronics, which are capable of bridging the elastomer flexure creases. It is essential that the electrical traces can not only bend while the tiles are folding, but also stretch to achieve compound folds. This presents a significant challenge which is not achievable with standard flexible circuits. As a solution, we pattern the trace with a series of cuts, or a mesh, that allows the metal to lengthen as needed (Fig. 1C). The meshes are micromachined from a sheet of copper-laminated polyimide in the exact pattern needed and bonded to the surface of the tiling. The actuators are soldered to the traces forming groups of actuators, as determined by the planning algorithm, to form a single power circuit. Thus to fold a shape, current is applied through one or more groups, heating all actuators in this phase through Joule heating until folds are complete and magnets engage. Then a second set of groups, or phase, is activated, a third, etc.

Programming. An important aspect of creating programmable matter is the actual programming that allows a user to trigger the formation of the desired shape. For the self-folding sheet, we developed a method for programming shapes by stickers. The idea is to enable users to request the formation of a shape without the use of a computer. The intuition is to create the smart sheets to include all the electronic circuitry except for actuator and connector wiring. Stickers are thin materials that contain the circuitry required to connect and trigger the correct actuators for making a specific object. If we have k objects that are achievable from a base sheet, the user will receive the base sheet and k stickers, each of which programs the formation of one of the k

*A compound fold is a fold that creases two or more layers of the sheet at once. These multiple layers are created through previous folds, see Fig. 1F.

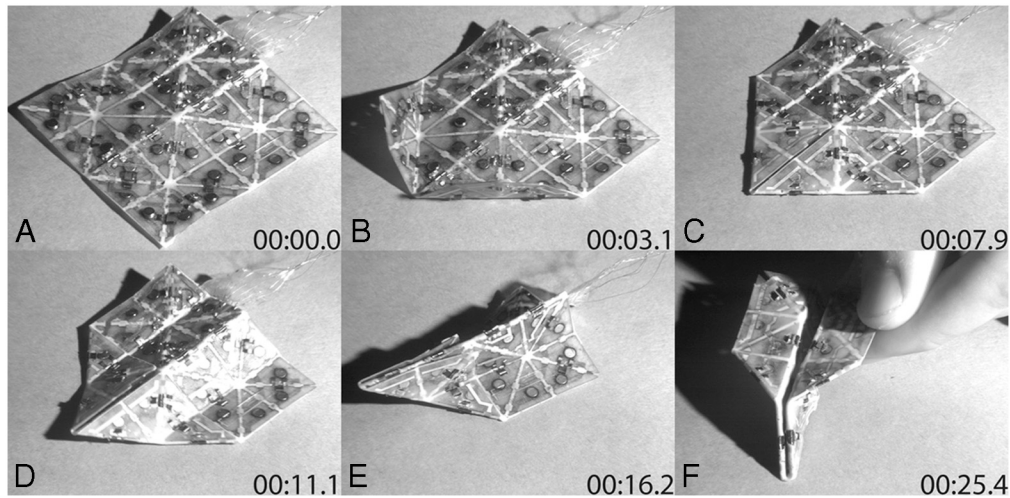


Fig. 5. Flat sheet prior to folding (A). Four-actuator group controlling flaps activated (B). Magnets for the first fold engaged (C). Remaining actuators are activated (D). Final shape (E) and inverted (F).

fabrication and performance. For fabrication, similar actuators could be reduced to millimeter feature sizes (26) using current methods or submillimeter scales using physical vapor deposition and patterning (27). For these actuator types, Joule heating becomes more effective at smaller scales, bandwidth will increase (mechanically as well as thermodynamically), and the energy density should remain constant. However, there are other potential actuation modes for smaller scales including pneumatic, electrostatic, and a variety of electroactive polymers (ionic and dielectric).

We have shown that the gaps between theory and practice can be surmounted for the shapes we have considered. Nonetheless the gaps suggest additional theory that would be interesting to develop if possible. For example, can we quantify the flexibility of the material needed by motions of an unfolded sheet into target folded states? The area of rigid origami, which forbids flexing except at creases, is relatively underdeveloped, making this a challenging question. For a given resolution, we could also study whether it is always possible to reach a folded state without

unfolding any folds, which would provide theoretical backing for potential wells.

Nevertheless, the current results suggest very promising applications. We can create a bulk material that is preprogrammed to adopt several shapes. Reconfigurable devices are an immediate application. Examples include a measuring cup system that folds to hold any amount from a quarter teaspoon to multiple cups, a shelving system that folds to fit a user-defined space with a user-defined number and type of divisions, and a puckering sheet that provides tactile feedback for displaying information to the blind or people in the dark. A wider range of devices could be incorporated to make a “Swiss army knife” of sorts, able to form a tripod, wrench, antenna, or splint. In the future, finer resolution and larger sheet sizes will become available as our manufacturing techniques evolve.

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